Exercise 7C: Solutions

1 **a** A and C (SAS)

b All of them (AAS)

c A and B (SSS)

2 **a** $\triangle ABC \equiv \triangle CDA$ (SSS)

b $\triangle CBA \equiv \triangle CDE$ (SAS)

c $\triangle CAD \equiv \triangle CAB$ (SAS)

d $\triangle ADC \equiv \triangle CBA$ (RHS)

e $\triangle DAB \equiv \triangle DCB$ (SSS)

f $\triangle DAB \equiv \triangle DBC$ (SAS)

3

Let AM be the bisector of $\angle CAB$.
Then

$AC = AB$ (Definition of isosceles)
$\angle CAM = \angle BAM$ (Construction)
$AM = AM$ (Common)
$\triangle ACM \equiv \triangle ABM$ (SAS)
$\therefore \angle ACM = \angle ABM$

That is $\angle ACB = \angle ABC$

4

Let AM be the bisector of $\angle CAB$.
Then

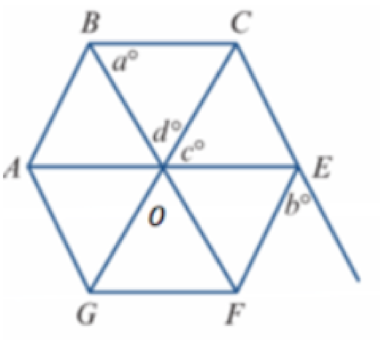
$$\begin{aligned} \angle ACM &= \angle ABM && \text{(Given)} \\ \angle CAM &= \angle BAM && \text{(Construction)} \\ AM &= AM && \text{(Common)} \\ \triangle ACM &\equiv \triangle ABM && \text{(AAS)} \\ \therefore AC &= AB \end{aligned}$$

5

$2\alpha + 2\beta = 360^\circ$
(Angle sum of quadrilateral)
$\therefore \alpha + \beta = 180^\circ$
Hence, cointerior angles are supplementary.
Therefore, $AB \parallel DC$

6 **a** $a = b = c = d = 60^\circ$

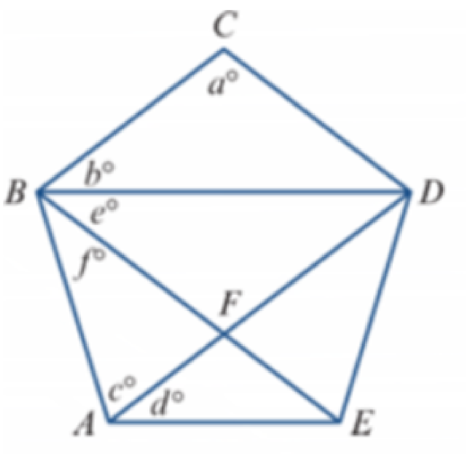
b



$\angle CB0 = \angle BOA = 60^\circ \therefore BC \parallel AE$ (alternate angles equal) Similarly $BE \parallel BA$

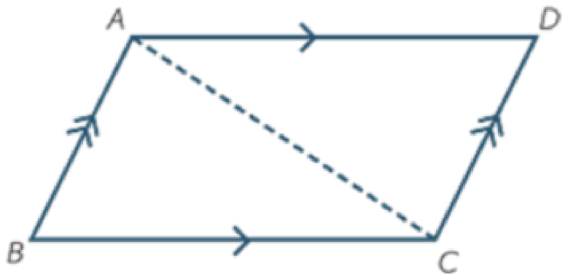
7 **a** $a = 108^\circ,\ b = 36^\circ,\ c = 72^\circ,\ d = 36^\circ,\ e = 36^\circ,\ f = 36^\circ$

b



$c^\circ + d^\circ = 108^\circ$ and $e^\circ + f^\circ = 72^\circ$
$\therefore BD \parallel AE$ (co-interior angles supplementary)
$b^\circ + e^\circ = 72^\circ$ and $a^\circ = 108^\circ$
$\therefore BE \parallel CD$ (co-interior angles supplementary)

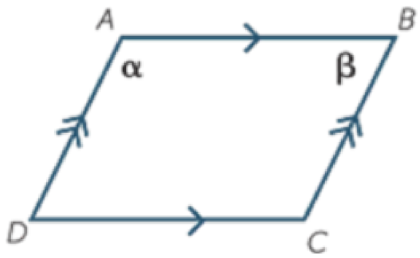
8 a



First prove opposite sides are equal.
$ABCD$ is a parallelogram, $AD \parallel BC$ and $AB \parallel DC$
Join diagonal AC
In $\triangle ABC$ and $\triangle CDA$

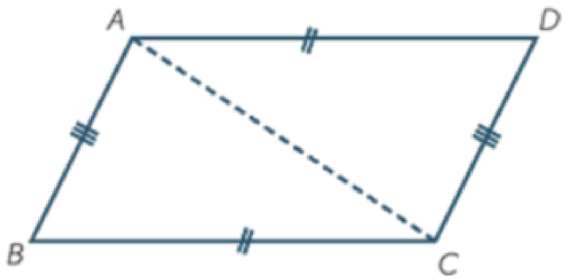
$\angle BAC = \angle DCA$ (alternate angles, $AB \parallel DC$)
$\angle BCA = \angle DAC$ (alternate angles, $AD \parallel$ to BC
$AC = CA$ (common)
$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)
$\therefore AB = CD$ and $AD = BC$

To prove opposite angles are equal.



Let $\angle DAC = \alpha$ and $\angle ABC = \beta$
$\alpha + \beta = 180^\circ$ (co-interior angles, $AD \parallel BC$)
$\therefore \angle ADC = \beta$ (co-interior angles, $AB \parallel DC$)
$\therefore \angle BCD = \alpha$ (co-interior angles, $AB \parallel DC$)

b



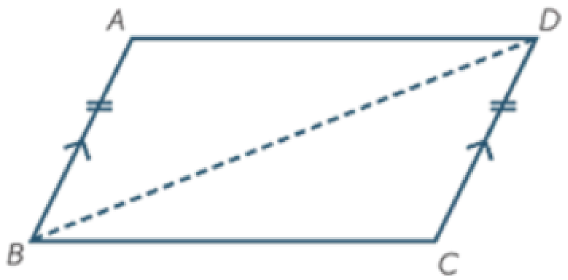
Join diagonal AC
In $\triangle ABC$ and $\triangle CDA$

$AD = CB$ (opposite sides equal)
$AB = CD$ (opposite sides equal)
$AC = CA$ (common)
$\therefore \triangle ABC \equiv \triangle CDA$ (SSS)
$\therefore \angle BAC = \angle DCA$
$\therefore AB \parallel DC$ (alternate angles equal
Furthermore,
$\therefore \angle DAC = \angle BCA$
$\therefore AD \parallel BC$ (alternate angles equal

c

From the diagram ,
$2\alpha + 2\beta = 360°$ (angle sum of quadilateral)
$\therefore \alpha + \beta = 180°$
Co-interior angles are supplementary.
$\therefore AB \parallel DC$ and $AD \parallel BC$

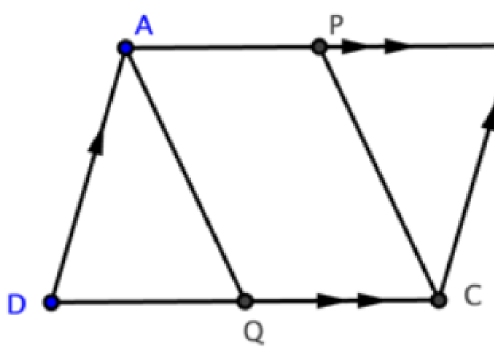
d



In $\triangle ABD$ and $\triangle CDB$

$AB = DC$ (given
$\angle ABD = \angle CDB$ (alternate angles
$BD = DB$ (common)
$\therefore \triangle ABD \equiv \triangle CDB$ (SAS)
$\therefore AD = BC$
$ABCD$ is a parallelogram

9



To prove:

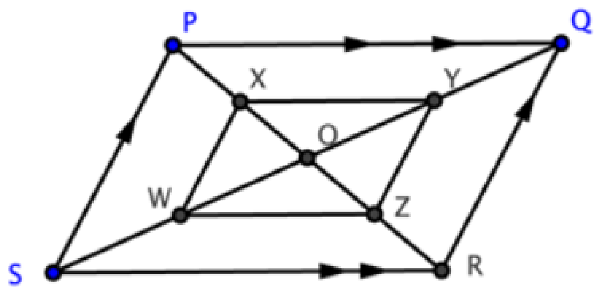
$APCQ$ is a parallelogram.

In $\triangle ADQ$ and $\triangle BPC$

$$AD = CB \text{ (opposite sides of a parallelogram)}$$
$$\angle D = \angle B \text{ (opposite angles of a parallelogram)}$$
$$DQ = BP \text{ (construction)}$$
$$\therefore \triangle ADQ \equiv \triangle CBP \text{ (SAS)}$$
$$\therefore AQ = PC$$

$\therefore$ $APCQ$ is a parallelogram (opposite sides are equal in length)

10



To prove:

$APCQ$ is a parallelogram.

The diagonals of a parallelogram bisect each other.

$\therefore XO = OZ$ and $WO = OY$

$\angle XOY = \angle WOZ$ and $\angle XOW = \angle YOZ$

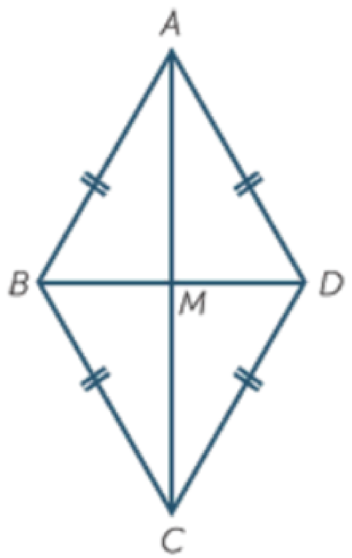
$\therefore \triangle XOY \equiv \triangle WOZ$ and $\triangle XOW \equiv \triangle YOZ$

$\therefore XY = WZ$ and $WX = ZY$

$\therefore$ $XYZW$ is a parallelogram (opposite sides of equal length)

11 A rhombus is defined as a parallelogram with a pair of adjacent sides equal in length. Therefore all the sides are equal in length. You should also prove that if a quadrilateral has all sides of equal length then it is a rhombus.

a



$$\triangle ABC \equiv \triangle ADC \text{ (SSS)}$$
$$\therefore \angle BAC = \angle DAM$$
$$\therefore \triangle ABM \equiv \triangle ADM \text{ (SAS)}$$
$$\therefore \angle BMA = \angle DMA = 90^\circ$$

(equal and supplementary)

$\therefore AC \perp BD$

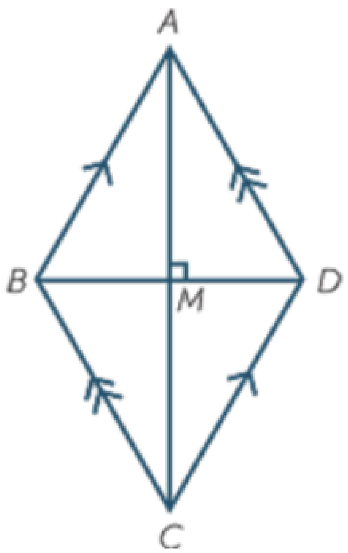
b Refer to the diagram for a

$\triangle ABC \equiv \triangle ADC$ (SSS)

$\therefore \angle BAC = \angle DAM$

Similarly for the other vertex angles

c



In $\triangle ABM$ and $\triangle CDM$

$$AM = MC \text{ (diagonals bisect each other)}$$
$$MM = DM \text{ (diagonals bisect each other)}$$
$$\angle BMA = \angle CMD = 90^\circ \text{ (diagonals are perpendicular)}$$
$$\therefore \triangle ABM \equiv \triangle CDM (\text{SAS})$$
$$\therefore AB = CD \text{ and } \angle MCD = \angle MAB$$
$$\therefore AB \parallel DC \text{ (alternate angles equal)}$$

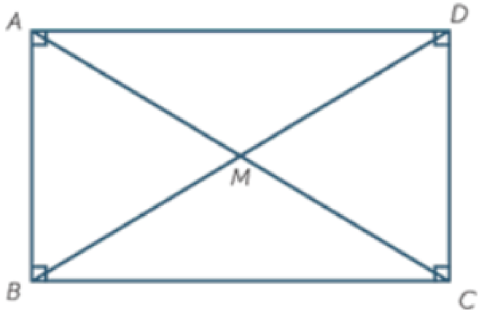
Similarly $BC = AD$ and $BC \parallel AD$

Finally $\triangle ABM \equiv \triangle CDM (\text{SAS})$

Hence $AB = AD$

We note that a shorter proof is available but we have proven several properties of rhombuses on the way through.

12a



$\triangle ABC \equiv \triangle DCB$ (SAS)

$\therefore AC = BD.$

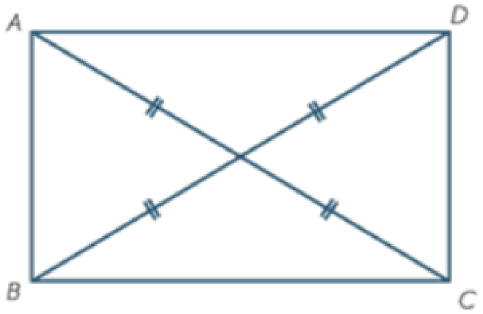
$ABCD$ is a rectangle and therefore a parallelogram

$\therefore$ diagonals bisect each other

b If a parallelogram has one right angle then:

the opposite angle is a right angle (opposite angles equal in a parallelogram). the cointerior angles are right angles.

c

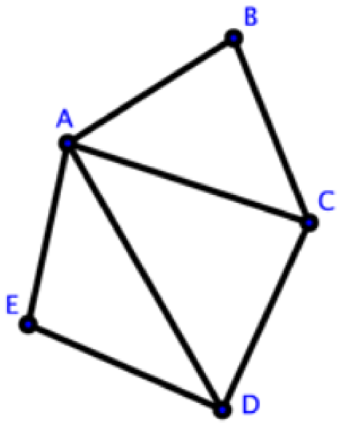


Let M be the pont of intersection of the diagonals.

$$\triangle AMD \equiv \triangle BMC(\text{SAS})$$
$$\triangle AMB \equiv \triangle DMC(\text{SAS})$$
All of these triangles are isosceles
$$\therefore \angle BAM = \angle DCM$$
$$\therefore AB \parallel DC$$
Similarly $AD \parallel BC$
$$\angle A = \angle B = \angle C = \angle D$$

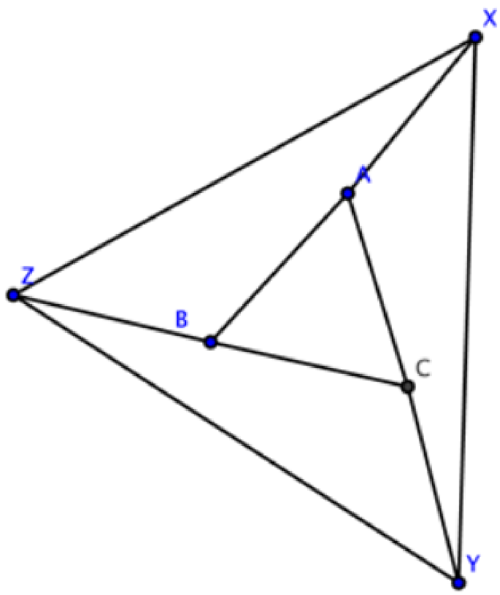
Therefore all right angles. Hence $ABCD$ is a rectangle.

13



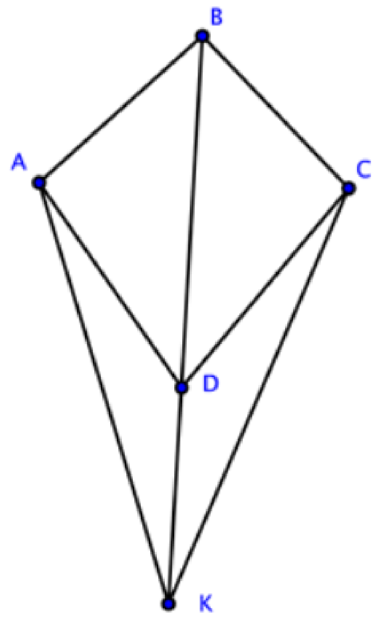
$\triangle ABC \equiv \triangle AED$ (SSS)
$\therefore \angle ABC = \angle AED$

14



$\triangle ZBX \equiv \triangle XAY \equiv \triangle YCZ$ (SAS)
$\therefore ZX = XY = YZ$

15

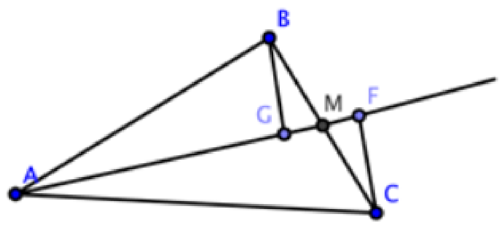


$\triangle ABD \equiv \triangle CBD$ (SSS)
$\therefore \angle ABD = \angle CBD$
$\therefore \triangle ABK \equiv \triangle CBK$ (SAS)
$\therefore AK = CK$

16 $\angle C = \angle A + \angle B$ implies that $\angle C = 90^\circ$.
$\triangle ABC$is a right-angled triangle.
Choose point D to complete the rectangle $ABCD$.
The rectangle has diagonals AB and CD which are of equal length and bisect each other.
Let M be the midpoint of AB.
Then $AB = 2CM$.

17 Let $\angle MNO = \angle MON = x^\circ$
Then $\angle ANO = (90 - x)^\circ$ and $\angle NMO = (180 - 2x)^\circ$

18



M is the midpoint of BC.
BG and CF are perpendicular to the median AM extended
$\triangle BMG \equiv \triangle CMF$ (ASA)
$\therefore BG = CF$